

# Determination of the Thiele - Small Parameters Using Two-Channel FFT Analysis

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## Abstract

The low-frequency performance of a driver for use in a direct-radiator loud-speaker system is characterized by its Thiele-Small parameters. Two impedance measurements are required for the calculation of these parameters. A fast technique for performing the impedance measurements is presented and the calculation of the basic parameters is reviewed. Computer implementation for numerical calculation and control of the analyzer is discussed and the results for a typical driver are shown.

## Introduction

The parameters introduced by Mr. Neville Thiele and Dr. Richard H. Small have become an industry standard in the analysis and design of both closed and ported loudspeaker enclosures. Since the first publishing of Dr. Small's work, the sophistication of electronic measuring equipment has advanced tremendously, most notably with the introduction of the FFT analyzer. Digital microprocessor technology has increased the speed and precision of laboratory measurements and has enabled the user of this equipment to perform a variety of tasks not possible fifteen years ago.

Two-channel FFT analysis involves the measurement of the input and output of a linear system to determine its frequency response or transfer function. By using the method described by Small,<sup>[1]</sup> in which an impedance measurement is made on a free-standing driver, followed by a second measurement on the same driver placed in a sealed, unlined test enclosure of known volume, a great deal of information about the driver's characteristics can be determined. The fol-

lowing is a list of parameters this method will determine:

$R_E$	DC resistance of driver*	
$F_S$	resonant frequency of driver	
$Z_{\text{max}}$	impedance at resonance	
$Q_{MS}$	non-electrical driver Q	
$Q_{ES}$	electrical driver Q	
$Q_{TS}$	total Q of driver	
$V_{AS}$	compliance equivalent volume	
$C_{AS}$	acoustic compliance of driver suspension	
$M_{AS}$	acoustic mass of driver diaphragm assembly (including air load)	
$R_{AS}$	acoustic resistance of driver suspension losses	
$BL$	magnetic flux density length product	- coil
$\eta_0$	acoustic reference efficiency	

\* The DC resistance is typically measured with an ohm-meter

## Test Set-up

The circuit configuration for the impedance measurements is shown in Fig. 1. All measurements in this note were made using B&K Dual Channel Signal Analyzer Type 2032. The voltage  $V_s$  across the driver is the input to channel B of the analyzer, while the generator voltage  $V_g$  is the input to channel A. The analyzer calculates the transfer function by dividing the spectrum of channel B by the spectrum of Channel A ( $V_{\text{out}}/V_{\text{in}}$ ) and displaying the result.

The 1 k $\Omega$  resistor that appears between the generator and driver makes the generator appear as a constant-current source to the driver.<sup>[2]</sup> For  $|Z| \ll 1000$ .

$$I = V_g / 1000 \quad \text{then}$$

$$|Z| = V_s / I$$

$$= \frac{V_s}{V_g / 1000}$$

$$= \frac{1000 V_s}{V_g} \quad (1)$$

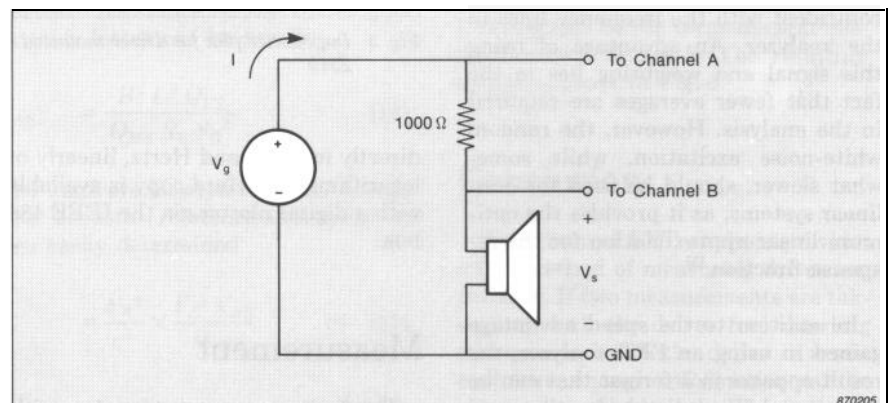


Fig. 1. Circuit configuration for impedance measurement

The scaling factor of 1000 is easily incorporated into the measurement via the front panel of the analyzer. Alternatively, a  $V$  to  $I$  converter (transconductance amplifier) could be used. For most drivers, however, the resistor circuit is quite sufficient.

The output resistance of the generator can also affect the impedance measurement, as this resistance is in series with the  $1\text{ k}\Omega$  resistor and the voice coil resistance. If the output voltage of the generator does not drop under load conditions, then the generator output resistance may be considered to be zero with respect to its effect on the impedance measurement.<sup>[1]</sup>

## Input Source

Traditionally, swept-sine measurements have been used to determine the response versus frequency of a linear system. This method, while accurate, is very slow. The Dual Channel Signal Analyzer Type 2032 used in these measurements has the option of a built-in random noise generator. A choice is available of either Gaussian white noise, or pseudo-random noise. One advantage of using a random signal at the input is that it can be filtered to suit the particular frequency range of interest. Therefore, it will not excite the system outside the analysis bandwidth. This, in turn, increases the dynamic range of the measurement.<sup>[3]</sup>

When using white noise as an input excitation, a Hanning weighting function is required, due to the continuous nature of the signal. A pseudo-random signal can be used in conjunction with a rectangular weighting to speed up the analysis. The pseudo-random signal consists of a random signal of finite period that is synchronized with the record time of the analyzer. It yields discrete frequency components coincident with the frequency lines in the analyzer. An advantage of using this signal and weighting lies in the fact that fewer averages are required in the analysis. However, the random white-noise excitation, while somewhat slower, should be used for non-linear systems, as it provides the optimum linear approximation for the response function.<sup>[3]</sup>

In addition to the speed advantage gained in using an FFT analyzer, the result appears in a format that can be interpreted immediately by the user. The analyzer can be scaled to read out

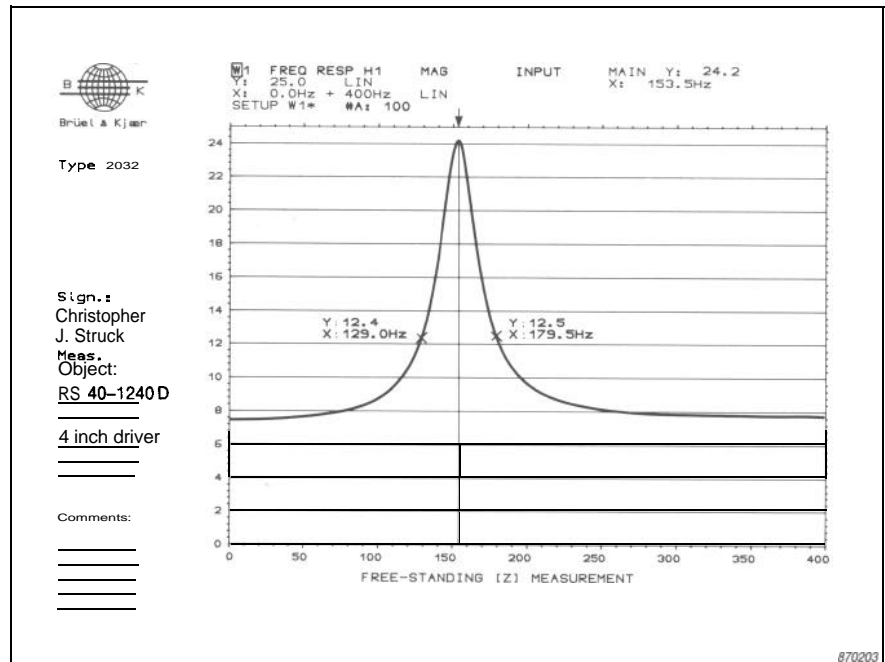


Fig. 2. Impedance plot for free-standing driver. Plot made with Graphics Plotter Type 2319

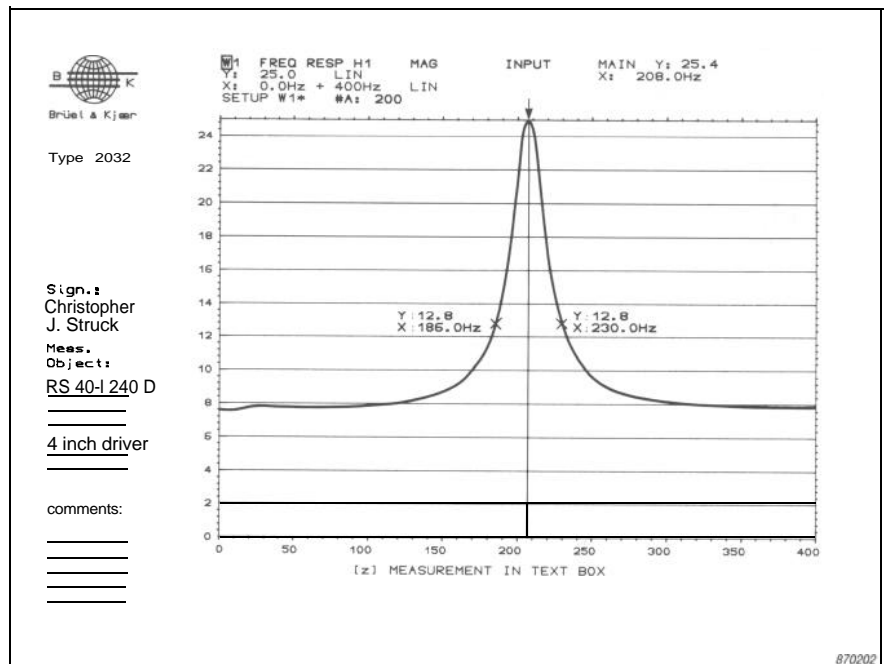


Fig. 3. Impedance plot for driver mounted in test box. Plot made with Graphics Plotter Type 2319

directly in ohms and Hertz, linearly or logarithmically. Hard copy is available with a digital plotter via the IEEE 488 bus.

## Measurement

The first measurement is taken with the driver free-standing. The imped-

ance plots obtained will have the characteristic shape seen in Fig. 2. Care must be taken to avoid blocking the vents in the rear of the magnet structure, if the driver is so equipped. During the measurement, the driver may be placed face-up on a work bench or test table, provided this structure is rigid and massive enough not to contribute to the mechanical resonance of the driver.

The resonant frequency,  $F_S$ , of the driver is found where the impedance is a maximum ( $Z_{max}$ ). If  $R_E$  is the DC resistance of the driver, then according to Small,<sup>[3]</sup> the ratio of the maximum voice-coil impedance to the DC resistance is defined as

$$r_o = Z_{max} / R_E \quad (2)$$

Two frequencies,  $F_{1S} < F_S$  and  $F_{2S} > F_S$ , can be found on the impedance plot where

$$|Z| = \sqrt{r_o} R_E \quad (3)$$

then

$$Q_{MS} = \frac{F_S \sqrt{r_o}}{F_{2S} - F_{1S}} \quad (4)$$

and

$$Q_{ES} = \frac{Q_{MS}}{r_o - 1} \quad (5)$$

The measurement procedure is then repeated with the driver mounted in a sealed, unlined test box, in order to add a known compliance to the system. An upward shift in the resonant frequency is observed as shown in Fig. 3. The new resonant frequency is designated  $F_{CT}$ , and the corresponding frequencies  $F_{1CT}$  and  $F_{2CT}$  are obtained using the same procedure described above. Then,

$$Q_{MCT} = \frac{F_{CT} \sqrt{r_o}}{F_{2CT} - F_{1CT}} \quad (6)$$

and

$$Q_{ECT} = \frac{Q_{MCT}}{r_o - 1} \quad (7)$$

## Calculations

The remaining parameters can be calculated from the previously measured data. From Small<sup>[11]</sup>:

$$Q_{TS} = \frac{Q_{MS} Q_{ES}}{Q_{MS} + Q_{ES}} \quad (8)$$

and

$$V_{AS} = V_T \left[ \frac{F_{CT} Q_{ECT}}{F_S Q_{ES}} - 1 \right] \quad (9)$$

where  $V_T$  is the internal volume of the test box.  $V_T$  and  $V_{AS}$  are generally expressed in liters. From this, the acoustic compliance of the driver can be determined

$$C_{AS} = \frac{V_{AS}}{1000 \rho_o c^2} \quad (10)$$

where  $\rho_o$  is the density of air and  $c$  is the velocity of sound, the factor of 1000 converts  $V_{AS}$  from liters to cubic meters. So

$$C_{AS} = \frac{V_{AS}}{1,41 \times 10^8} \quad (11)$$

The units are meters<sup>5</sup>/Newton. The acoustic mass of the driver can be related to the compliance with the resonant frequency<sup>[4]</sup>

$$M_{AS} = \frac{1}{4 \pi^2 F_S^2 C_{AS}} \quad (12)$$

Here, the units are kilograms/meter<sup>4</sup>. The  $BL$  product can also be calculated from these parameters. First, we calculate the acoustic mass of the driver in the test box,

$$M_{ACT} = M_{AS} \frac{F_S^2}{F_{CT}^2} \left[ 1 + \frac{V_{AS}}{V_T} \right] \quad (13)$$

Then, let  $S_o$  be the effective surface area of the driver. This is calculated by measuring the diameter,  $d$ , of the driver from the middle of the surround to the middle of the surround. Then

$$S_D = \pi \frac{d^2}{4} \quad (14)$$

and

$$BL = \sqrt{\frac{2 \pi F_{CT} R_E M_{ACT} S_D^2}{Q_{ECT}}} \quad (15)$$

measured in Webers/meter. The acoustic resistance of the driver suspension losses can be found

$$R_{AS} = \frac{B^2 L^2 Q_{ES}}{Q_{MS} R_E S_D^2} \quad (16)$$

The units are Newton-seconds/meter<sup>5</sup>. The acoustic reference efficiency is also easily determined

$$\eta_o = \frac{4 \pi^2}{c^3} \times \frac{F_S^3 V_{AS}}{Q_{ES}} \quad (17)$$

For  $V_{AS}$  in liters

$$\eta_o = 9,6 \times 10^{-10} \frac{F_S^3 V_{AS}}{Q_{ES}} \quad (18)$$

This number is usually multiplied by 100 and expressed in percent.

## Application

A tuned closed or ported box system can be designed using these parameters. The low-frequency response can be predicted by varying the volume of the box to be designed to obtain the desired alignment.<sup>[5]</sup> The reference efficiency is used to match drivers of similar efficiency in a multi-way system.  $C_{AS}$ ,  $M_{AS}$  and  $R_{AS}$  can be used for equivalent circuit modelling. The  $BL$  product is used in the design and quality control of drivers.

## Computer Implementation

Even with a hand calculator, the calculations involved in this analysis can become tedious. Computer implementation is attractive for several reasons. The calculations can be performed much more rapidly. If the 1000  $\Omega$  resistor is used in the measurement, a correction can be applied to the impedance measurement to obtain exact values for the calculations. A printer can provide hard-copy for the results, or the results can be stored on a floppy disc for use at a later date. The computer can act as a control device via the IEEE 488 bus to automate the test procedure. A plotter can be used in conjunction with the analyzer to obtain very fast impedance plots. A flow-chart for the control/calculation program appears in Fig. 4. Parameters for the test driver were calculated using the impedance plots in Figs. 2 and 3. A diagram of the measurement setup is shown in Fig. 5. The resulting output appears in Fig. 6.

## Conclusion

Two-channel FFT analysis provides a fast method of measuring driver impedance. If two measurements are taken, one with the driver free-standing and a second with the driver in a test enclosure, enough information can be obtained to calculate the Thiele-Small parameters. The speed of this tech-

nique can be increased by using a computer in conjunction with the analyzer to automate the measurement and perform the calculations.

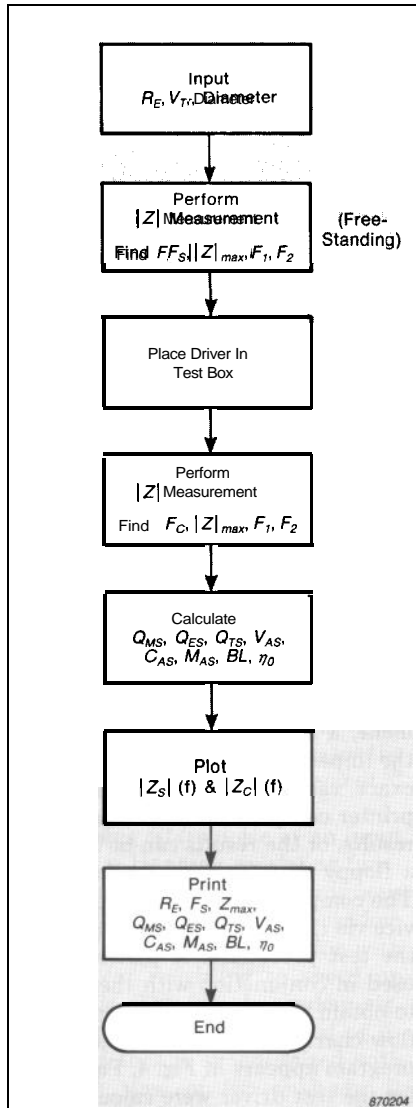


Fig. 4. Flow chart for control/calculation program.

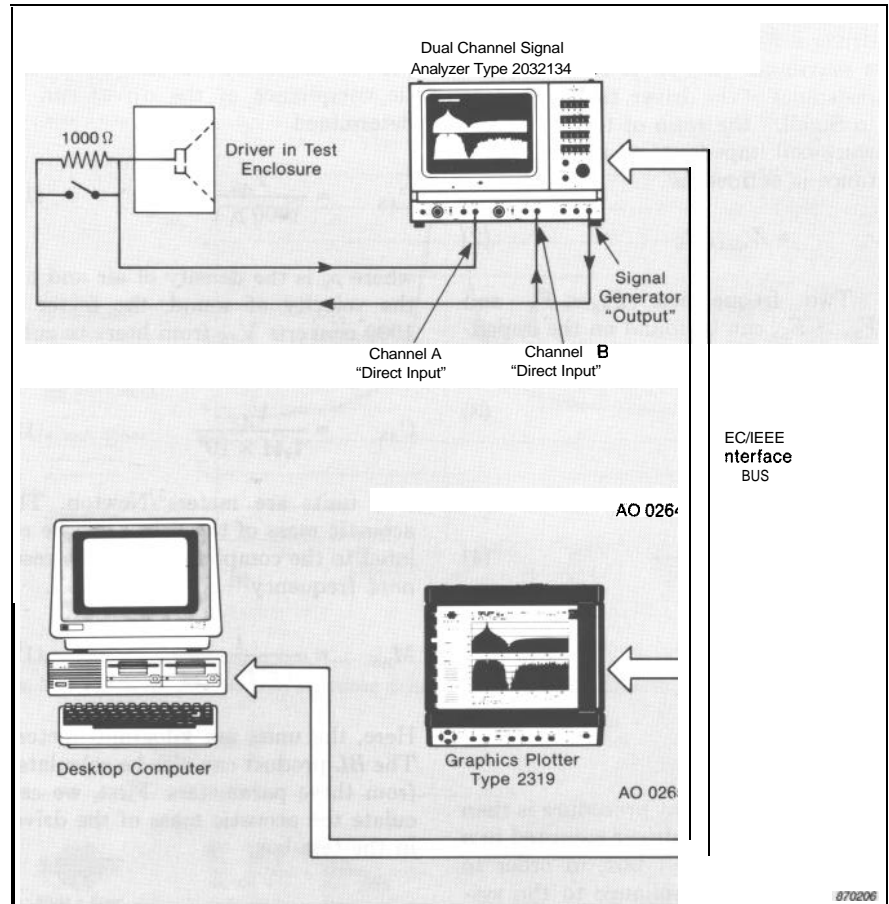


Fig. 5. Diagram of FFT-based measurement system for measuring Thiele-Small parameters

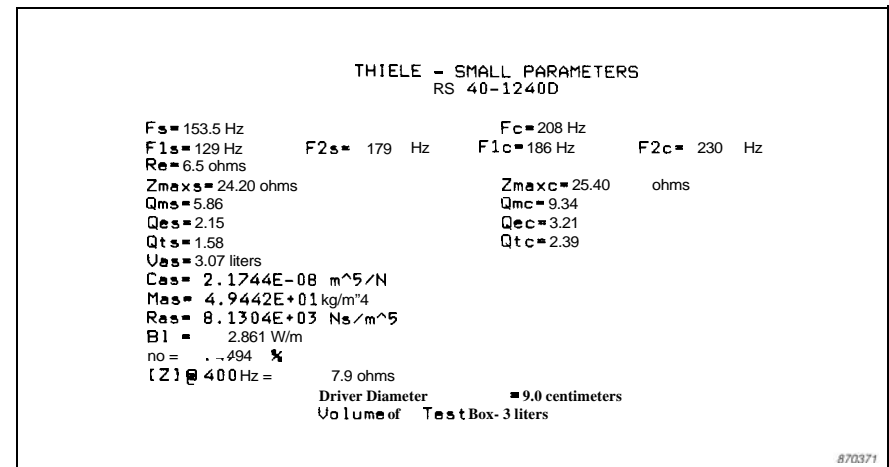


Fig. 6. Computer output of Thiele-Small parameters for 4-inch test driver

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